THE MONTE CARLO METHOD

1 Simulate 40 throws with a red and a white die.

a) What is the probability that the sum of the pips is 2?

b) What is the probability that the sum of the pips is 5?

1st throw	2nd throw	sum
3	1	4
6	1	7
1	4	5
6	6	12
2	2	4
1	1	2
5	1	6
3	2	5
6	1	7
6	4	10

1st throw	2nd throw	sum
2	6	8
5	2	7
2	6	8
6	1	7
1	5	6
6	4	10
2	1	3
6	3	9
2	1	3
2	6	8

1st throw	2nd throw	sum
3	4	7
4	1	5
1	1	2
5	5	10
6	3	9
1	3	4
6	5	9
4	4	8
6	5	11
5	4	9

1st throw	2nd throw	sum
3	2	5
4	6	10
1	3	4
3	2	5
5	6	11
3	3	6
2	4	6
4	5	9
5	3	8
6	4	10

a) The sum 2 occurs twice.

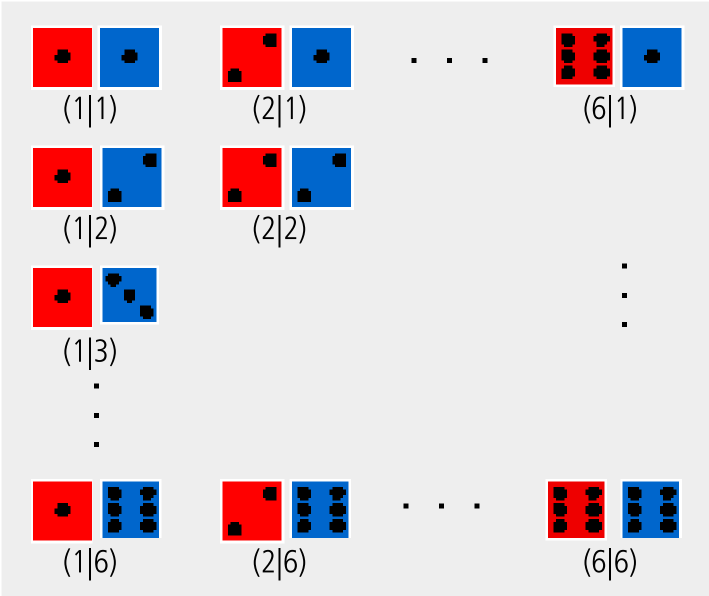
$$p(\text{"sum} = 2\text{"}) \approx \frac{2}{40} = \frac{1}{20} = \mathbf{0.05}$$

b) The sum 5 occurs 5 times.

$$p(\text{"sum} = 5\text{"}) \approx \frac{5}{40} = \frac{1}{8} = \mathbf{0.125}$$

$\boxed{2}$ Calculate theoretically the probabilities of $\boxed{1}$.

$$p(\text{event } A) = \frac{\text{number of outcomes in which A occurs}}{\text{number of possible outcomes}} = \frac{|A|}{|\Omega|}$$



a) With a throw with a red and a white die the outcome is a pair of numbers, both of them between 1 and 6, and the first number representing the result of the red die, the second one the result of the white die.
The sample space:

$$\Omega = \{(1|1), (1|2), (1|3), \ldots, (6|6)\} \quad \Rightarrow \quad n(\Omega) = 36$$

For the event A = "sum of the pips = 2":

$$A = \{(1|1)\} \quad \Rightarrow \quad n(A) = 1 \approx 0.028$$

$$\Rightarrow \quad p(A) = \mathbf{\frac{1}{36}}$$

b) For the event B = "sum of the pips = 5":

$$B = \{(1|4), (4|1), (2|3), (3|2)\} \quad \Rightarrow \quad n(B) = 4$$

$$\Rightarrow \quad p(B) = \frac{4}{36} = \mathbf{\frac{1}{9}}$$

3

Each Sunday morning the same five people come to the newsagent's at the railway station to fetch the newspaper. They arrive independently at any time between 9.00 and 9.59 and stay exactly 1 minute to have a chat with the newsagent.
What is the probability that at least two of these five customers meet?
Solve with the Monte Carlo method and simulate 20 Sundays.

The arrival time (in minutes after 9 o'clock) of each person can be generated with rand()*59. If two or more arrival times are less than 1 minute apart these people meet. So 20 Sundays can be simulated and the result be noted down.

Sunday	1st person	2nd person	3rd person	4th person	5th person	people meet
1	41.162	46.706	21.761	29.908	28.712	no
2	25.465	58.298	13.990	28.735	4.190	no
3	13.707	6.244	56.679	13.135	9.237	no
4	13.254	32.830	15.340	43.377	20.720	no
5	9.061	46.780	25.193	**35.02**	**34.11**	yes
6	**6.947**	**7.198**	31.787	14.136	5.259	yes
7	16.168	48.728	14.844	50.540	41.953	no
8	47.380	8.818	0.331	58.691	6.376	no
9	55.919	42.325	54.782	35.819	8.441	no
10	43.747	**57.4**	48.632	**57.84**	4.916	yes
11	54.119	46.705	23.509	10.308	27.025	no
12	57.351	18.862	29.581	23.896	0.941	no
13	55.645	**22.94**	**22.75**	36.800	4.360	yes
14	43.022	23.815	31.888	8.160	58.777	no
15	9.150	22.727	16.757	6.531	24.276	no
16	14.167	**55.26**	31.252	16.871	**54.41**	yes
17	49.872	21.073	13.344	30.601	47.933	no
18	30.433	2.947	48.584	5.385	52.736	no
19	33.218	38.686	45.719	1.187	52.618	no
20	52.860	30.019	20.659	56.753	10.106	no

In 5 Sundays out of 20 at least two people meet at the station.

$$\Rightarrow \qquad p(B) \approx \frac{5}{20} = \mathbf{0.25}$$

$\boxed{4}$ Assuming there are exactly 99,999 cars in a canton with number plates from 1 up to 99,999.
What are the chances that a randomly chosen number plate shows at least one 5?
Solve with the Monte Carlo method and simulate 20 number plates.

1-5	5-10
85'559	5'259
97'709	72'238
27'831	1'257
27'522	42'112
12'180	30'764

10-15	15-20
97'221	98'467
2'882	72'454
83'782	30'175
61'813	24'928
20'509	94'619

RandInt(1,99999)

20 randomly produced number plates are checked wether they show at least one 5:

$$p(A) \approx \frac{7}{20} = \mathbf{0.35}$$

$\boxed{5}$ Solve $\boxed{4}$ theoretically.

$|\Omega|$ is clear: $|\Omega| = 99{,}999$

A = "shows at least one 5".
$|A|$ is not so clear. It is much easier to look at the contrary:

$\overline{A}$ = "the number plate shows no 5".

$|\overline{A}| = 9^5$

9 9 9 9 9 possibilities

$\Rightarrow \quad |A| = 99{,}999 - 9^5 = 40{,}950$

$\Rightarrow \quad p(A) = \frac{40{,}950}{99{,}999} \approx \mathbf{41\,\%}$

6

On a mild Saturday morning at a lake five hunters shoot at the same time at a flight of five ducks. Each hunter chooses his prey at random and independent from the others, and each hunter is a perfect rifleman who hits the target under all circumstances.
What are the chances that exactly two ducks are killed on such a Saturday morning?
Solve with the Monte Carlo method and simulate 10 Sundays.

For each Saturday RandInt(1,5) generates a random integer for each hunter. That integer corresponds to the duck he is shooting at.

Saturday	1st hunter	2nd hunter	3rd hunter	4th hunter	5th hunter	ducks shot
1	1	4	3	2	2	**4**
2	4	1	3	5	1	**4**
3	2	4	1	2	1	**3**
4	5	4	2	3	4	**4**
5	2	1	3	1	2	**3**
6	4	5	4	1	2	**4**
2	3	2	1	5	5	**4**
8	3	3	4	2	4	**3**
9	3	1	5	3	2	**4**
10	5	2	3	5	4	**4**
11	3	1	5	5	3	**3**
12	1	4	4	2	5	**4**
13	2	5	5	4	5	**3**
14	4	1	5	3	1	**4**
15	3	1	3	4	4	**3**
16	2	2	2	2	3	**2**
17	3	2	4	3	3	**3**
18	3	2	2	3	5	**3**
19	5	2	3	4	2	**4**
20	4	4	2	2	2	**2**

$$\Rightarrow \quad p(\text{exactly 2 ducks arekilled}) \approx \frac{2}{20} = \frac{1}{10}$$